

Semi-classical black holes with large N re-scaling and information loss problem

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ABSTRACT: We consider semi-classical solutions of black holes and related re-scalings with large N massless fields to study the information loss problem. For a given semi-classical metric solution of a $N = 1$ universe, we can find another solution of a large N universe by using re-scaling. Here, after the re-scaling, any curvature quantity takes a sufficiently small value without changing its causal structure. A large number of massless fields is realized from large black holes in the brane world with many D3-branes and also from small black holes in the brane world. From re-scalings, we argue that black hole complementarity or violation of locality for semi-classical black holes cannot provide a fundamental resolution of the information loss problem. We claim that any fundamental resolution of the information loss problem should resolve the problem around singularity.

KEYWORDS: Black Holes, Black Holes in String Theory, D-branes.

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1. Introduction

The information loss problem of black holes was motivated by semi-classical calculations [1]. Applying quantum field theory in a classical metric of a black hole, it is observed that the black hole is evaporating by emitting Hawking radiations [2]. This calculation poses a very profound question to the unitarity of quantum mechanics. Even though we do not have the final answer due to the absence of quantum gravity, we can advance the problem by constructing and speculating upon different semi-classical black hole solutions.

Black hole complementarity is a typical example of such reasoning [3]. It reflects the non-locality which quantum gravity may contain in a certain form [4] and provides rich implications to be considered in constructing quantum gravity. However, its motivation is essentially semi-classical. According to black hole complementarity, after the information retention time [5] (when the initial area of a black hole decreases to its half value), an

observer outside of the black hole can see the information of the in-falling matter via the Hawking radiation. However, since the free-falling information is not affected by the Hawking radiation, two copies of information may exist; this appears to violate the no cloning theorem of quantum mechanics [6]. Black hole complementarity argues that this does not pose a problem, since these two copies cannot be observed by a single observer. In a Schwarzschild black hole, this assertion appears to hold well [7].

We introduce an interesting semi-classical setup to discuss the information loss problem: *semi-classical gravity with a large number of massless fields*. This kind of setup has already been considered to clarify semi-classical approximations for quantization of gravity. If there is a sufficiently large number of massless fields, they will be dominant over the gravitons, and can thereby justify semi-classical approaches [8][9]. However, in the context of whole quantum gravity, it is still questionable whether it is reasonable to use the limit of many massless fields. String theory will give a low energy effective action, which contains the gravity as well as some matter fields, and the action will allow semi-classical gravity in general. If string theory can allow a sufficiently large number of massless fields, then the large number setup will be helpful to understand the information loss problem [10], since any fundamental resolution of the information loss problem should be valid even in that extreme case.

It is not difficult to obtain a large number of massless fields in string theory; if there are many D-branes, there will be many massless fields. Of course, if there are many D-branes, the gravitational back-reactions should be strong. Thus, to justify the semi-classical gravity, the weak coupling limit should be used; the coupling should be increasingly smaller as the number of branes increases. This raises two potential problems: first, the gravitational constant becomes smaller and smaller, and the Planck mass becomes larger and larger; second, in order to obtain 4-dimensional semi-classical gravity, a compactification may be needed, which in general is not intuitively clear.

In this paper, these two potential concerns are answered. First, even though the gravitational constant is changed, if the Planck units are chosen as the fundamental units, the value of the gravitational constant itself will be meaningless *for a given universe*. Second, to justify the dimensional reduction, we use very well-known results from the brane world scenario [11][12]. If one assumes a warped geometry, in the low energy limit, the brane will resemble the normal 4-dimensions, even though the other dimensions are not completely compactified [12]. Here, some effects through the orthogonal directions should exist; but as we discuss, it is well known that the effect can be ignored in the low energy large black hole limit [13]. In fact, there are two major branches of research on the brane world scenario. The first is a small black hole, which may be obtained from LHC or future particle accelerators [14]. The second is a large black hole, which is greater than the horizon size of its bulk anti de Sitter space [15][13]. In this paper, we consider both scenarios.

We introduce re-scalings of semi-classical solutions between different universes: for a given semi-classical metric solution of a $N = 1$ universe, we find another solution of a large N universe by using re-scaling. Here, after re-scaling, any curvature quantity takes a sufficiently small value without changing its causal structure. In this context, we argue that black hole complementarity and violation of locality fail to provide a fundamental

resolution of the information loss problem.

This paper is organized as follows: first, we introduce re-scaling; second, we prepare a universe from the brane world setup that allows re-scaling. Therefore, the setup is believed to be allowed in string theory in principle. The conclusion is as follows: if we get a solution of the semi-classical equation with a small number of massless fields, we can find a universe where we can justify the same causal structure with reasonably small curvatures in a large number of massless fields limit. This result will have important physical implications, especially on black hole complementarity, violation of locality, the black hole remnant picture, and issues on singularity around the information loss problem.

2. Constants and units of nature

We can specify important fundamental constants and their dimensions, where L , M , and T are dimensions of length, mass, and time.

$$\begin{aligned} [c] &= LT^{-1} && \text{the speed of light,} \\ [G_4] &= L^3 M^{-1} T^{-2} && \text{the gravitational constant for 4-dimensions,} \\ [\hbar] &= L^2 M T^{-1} && \text{the Planck constant.} \end{aligned} \quad (2.1)$$

Also, we can fix the fundamental units by using the Planck units:

$$\begin{aligned} l_{Pl} &= \sqrt{\frac{\hbar G_4}{c^3}} && \text{the Planck length,} \\ m_{Pl} &= \sqrt{\frac{\hbar c}{G_4}} && \text{the Planck mass,} \\ t_{Pl} &= \sqrt{\frac{\hbar G_4}{c^5}} && \text{the Planck time.} \end{aligned} \quad (2.2)$$

Basically, we determine these constants or units by experiments, and we can define these constants as 1 in the Planck units.

Let us choose $c = 1$. This is related to consider relativistic physics. Then, $L = T$ is obtained. In order to study the relativistic quantum phenomena, it is convenient to choose $c = \hbar = 1$. $L = T = M^{-1}$ then holds, and the unit length and time are proportional to $\sqrt{G_4}$, while the unit mass is proportional to $1/\sqrt{G_4}$. In this approach, one chooses quantum mechanics first, and gravity is considered next.

If one considers gravity first and quantum mechanics second, however, it will be more convenient to choose $c = G_4 = 1$. *This is the basic philosophy of semi-classical gravity.* $L = T = M$ then holds, and the unit time, length, and mass are proportional to $\sqrt{\hbar}$. Since the scalar curvature R has a dimension L^{-2} , it is reasonable to think that the cutoff of curvature is on the order of l_{Pl}^{-2} . In the $\hbar \rightarrow 0$ limit, the cutoff becomes infinite; i.e., curvature singularity occurs where the curvature function becomes infinite. However, if \hbar is not zero, there is a finite cutoff; if a curvature function of a certain region becomes on the order of the cutoff, the causal structure of there cannot be justified in the semi-classical sense.

In this paper, we follow the semi-classical philosophy. We introduce re-scalings in this philosophy: for a given fixed causal structure, as the number of massless fields increases, the relative values up to the Planck length become increasingly larger. Then, effectively all curvature quantities will become smaller and smaller. Therefore, the causal structure will be justified with reasonably small curvatures.

3. Methods of large N re-scaling

3.1 The semi-classical theory

Let us assume that a low energy effective action is given as follows:

$$S = (4\text{-D gravity}) + (\text{a large number of massless fields}). \quad (3.1)$$

We consider back-reactions on the metric from the field dynamics,

$$g_{ab}^f = g_{ab} + \gamma_{ab}, \quad (3.2)$$

where the metric g_{ab}^f consists of two parts: g_{ab} is a classical metric of a background and γ_{ab} is contribution of the graviton [9]. For simplicity, let us assume that we make a black hole with a massless scalar field. (All of the arguments of this section can easily be applied to general classical field configurations.) The equation of motion for one field is as follows:

$$\phi_{;ab}g^{ab} = 0. \quad (3.3)$$

And the Einstein equation can be written as the expansion of \hbar :

$$G_{\mu\nu}^f = G_{\mu\nu} + G_{\mu\nu}^\gamma = 8\pi G_4(T_{\mu\nu} + N\hbar\langle T_{\mu\nu}^{(1)(1-loop)} \rangle) + \mathcal{O}(\hbar^2), \quad (3.4)$$

where $T_{\mu\nu}$ is the stress tensor for classical field configurations and $\langle T_{\mu\nu}^{(1)(1-loop)} \rangle$ is the 1-loop order re-normalized stress tensor of *one* massless field. Here, the energy-momentum tensor of the classical part becomes

$$T_{ab} = \phi_{;a}\phi_{;b} - \frac{1}{2}\phi_{;c}\phi_{;d}g^{cd}g_{ab}, \quad (3.5)$$

and N is the number of massless fields. As we consider a large number of massless fields, the gravitational back-reaction $G_{\mu\nu}^\gamma$ should be suppressed by $1/N$ [9]. Then, if we take terms in the first order of \hbar , we arrive at the following equation:

$$G_{\mu\nu} = 8\pi G_4(T_{\mu\nu} + N\hbar\langle T_{\mu\nu}^{(1)(1-loop)} \rangle). \quad (3.6)$$

This description is reasonable in the following sense. If we make a black hole with one field, we do not need the details of interactions between fields; thus, $T_{\mu\nu}$ can be chosen simply. However, if we wish to include the Hawking radiation, it is necessary to include the re-normalized stress tensor. In black hole physics, if a black hole is sufficiently large, then the contribution to the Hawking radiation becomes (approximately) independent for each field. Thus, we could multiply a factor N in front of the second term.¹

¹If each scalar fields couple to gravity only and do not interact each other, the form of $N\langle T \rangle$ is justified for all loop orders. If there are interactions between fields, and their effects are not negligible, then each interactions of n -loop order will be $\sim N^n \hbar^n$. Then, we can change Equation 3.6 as $G = T + N\hbar\langle T \rangle + (N\hbar)^2\langle T \rangle + \dots$; then, still we can apply the same scheme of the following subsection.

3.2 The scheme of re-scaling

Let us assume that L, M , and T (which are arbitrary quantities with length, mass, and time dimensions in the Planck units of the $N = 1$ universe) are solutions of the following equation:

$$G_{\mu\nu} = 8\pi(T_{\mu\nu} + \langle T_{\mu\nu} \rangle). \quad (3.7)$$

Now we define L', M' , and T' as follows:

$$\begin{aligned} L &= \frac{L'}{\sqrt{G_4 h}}, \\ M &= M' \sqrt{\frac{G_4}{h}}, \\ T &= \frac{T'}{\sqrt{G_4 h}}. \end{aligned} \quad (3.8)$$

Here, h is an arbitrary parameter that has the same dimension as \hbar ; G_4 is the gravitational constant for a large N universe. Then, since L, M , and T are solutions of Equation (3.7), L', M' , and T' are solutions of the following equation:

$$G_{\mu\nu} = 8\pi G_4(T_{\mu\nu} + h\langle T_{\mu\nu} \rangle). \quad (3.9)$$

This is easy to check: $G_{\mu\nu}$ has a dimension L^{-2} , $T_{\mu\nu}$ has a dimension ML^{-3} , and $\langle T_{\mu\nu} \rangle$ has a dimension L^{-4} in the one-loop order.

If there are N massless fields, the equation should take the form of Equation (3.6). Therefore, it is reasonable to choose $h = N\hbar$. Then, the physical length should be determined by the Planck unit, or the unit of \hbar :

$$\begin{aligned} L' &= \sqrt{G_4 N \hbar} L = (\sqrt{N} L) l_{Pl}, \\ M' &= \sqrt{\frac{N \hbar}{G_4}} M = (\sqrt{N} M) m_{Pl}, \\ T' &= \sqrt{G_4 N \hbar} T = (\sqrt{N} T) t_{Pl}, \\ R' &= \frac{R}{G_4 N \hbar} = \left(\frac{R}{N} \right) R_{Pl}, \end{aligned} \quad (3.10)$$

where R is a quantity that has a dimension of curvature. Now the re-scaled values are a set of solutions of Equation (3.6) [16].

In general, when there is a large number of massless fields, the gravitational coupling for 4-dimensions should become smaller and smaller. This phenomenon is discussed in the next section. However, this phenomenon itself does not contradict to the semi-classical physics, since the meaningful data is not the Planck scale itself, but the relative values up to the Planck scale.

Therefore, if we choose the Planck units of a large N universe, *whenever L, M , and T are solutions of Equation (3.7), $\sqrt{N}L, \sqrt{N}M$, and $\sqrt{N}T$ are solutions of Equation (3.6) with N massless fields.*

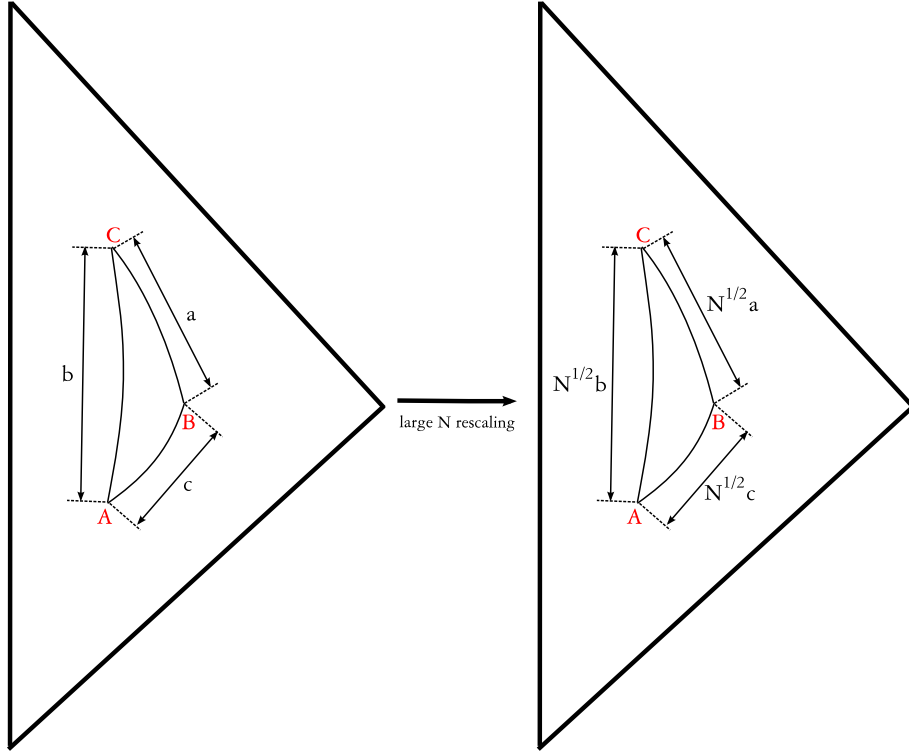


Figure 1: Invariance of the causal structure.

3.3 Invariance of causal structures

Note that, the causal structure is invariant up to this re-scaling, i.e., ds^2 is re-scaled to Nds^2 . For example, if the distance between points A and B along a time-like curve is c , B and C is a , and A and C is b (Figure 1), all of a , b , and c are re-scaled to $\sqrt{N}a$, $\sqrt{N}b$, and $\sqrt{N}c$. For simplicity, if one assumes a spherical symmetry (in the double null coordinate) $ds^2 = -\alpha(u, v)^2 du dv + r(u, v)^2 d\Omega^2$, the causal structure will be determined as $r(u, v)$, and all $r(u, v)$ will be re-scaled to $\sqrt{N}r(u, v)$; the causal structure is then invariant.

Two important remarks are noted here. First, the re-scaling conserves the causal structure of the metric. Therefore, *we can use the same Penrose diagram of the $N = 1$ case*. Second, if we can prepare a sufficiently large N universe, *even if a set of solutions has large curvature in the $N = 1$ case, we can find a universe where the curvature can be re-scaled to a sufficiently small value* by the re-scaling. Therefore, if one can prepare an arbitrary large N universe from string theory, any causal structure that is obtained to solve the semi-classical equation $G_{\mu\nu} = 8\pi(T_{\mu\nu} + \langle T_{\mu\nu} \rangle)$ is justified in string theory, since we can re-scale all curvatures to be small.

3.4 Conditions for re-scalings and applications to other theories

Now, one can specify the necessary conditions for re-scalings.

First, Equation (3.1) is needed.

Second, N should be sufficiently large.

Thus far, we have assumed that we construct a black hole with a massless field. However, in many cases, we can attempt to apply the same principles to other theories. For example, if one wants to discuss a charged black hole, it is convenient to assume a complex massless scalar field ϕ and a Maxwell field A_μ by

$$\mathcal{L} = -(\phi_{;a} + ieA_a\phi)g^{ab}(\bar{\phi}_{;b} - ieA_b\bar{\phi}) - \frac{1}{8\pi}F_{ab}F^{ab}, \quad (3.11)$$

where $F_{ab} = A_{b;a} - A_{a;b}$, and e is the unit charge. One can easily check that, e is re-scaled by e/\sqrt{N} . Also, if one wants to discuss a black hole with a potential, one can also impose the same scheme, for example, to the following potential:

$$V(\Phi) = A\Phi^4 + B\Phi^3 + C\Phi^2. \quad (3.12)$$

Here, each constants should be re-scaled by $1/N$. In both cases, we can apply the same re-scaling schemes, but some tunings of parameters may be required.

4. Realization of large N setup

We may try to find a large N setup directly from string theory. In fact, the KKLT scenario [17] is used to compactify 6-dimensions to a Calabi-Yau manifold, and some combinations of branes and anti-branes should be assumed. One may have to stack up D3-branes; the situation corresponds precisely with the setup that we wish to construct. Also, the cosmological constant can be fine-tuned by tuning the number of anti-D3-branes. Therefore, this scenario is the most natural estimation for our purposes.

However, there is a potential problem in applying this scheme. To control all orders of quantum effects, one needs to assume the tadpole cancelation condition:

$$\frac{\chi(X)}{24} = N_{D3} + \frac{1}{2\kappa_{10}^2 T_3} \int_M H_3 \wedge F_3, \quad (4.1)$$

where $\chi(X)$ is the Euler characteristic of a manifold X and N_{D3} is the number of net D3-branes. The question then is whether there exists a Calabi-Yau manifold, even in principle only, with such a large number of Euler characteristics to cancel out the large number of net D3-branes.

Rather to answer the question directly, we will now look at more concrete cases from the brane world. This is a less controversial way to obtain 4-dimensions from a large number of D3-branes.

4.1 Brane world: large black holes in weak energy limit

We assume there are \mathcal{N} D3-branes with a weak coupling limit. And, perpendicular directions are compactified by volume V_{10-D} . Here, $g^2 = \exp \langle \phi \rangle$ is the coupling (we follow the notations of [18]). We choose the following condition:

$$g^2 \mathcal{N} \lesssim 1. \quad (4.2)$$

Then, the number of fields will be on the order of \mathcal{N}^2 , since \mathcal{N} D-branes induces $SU(\mathcal{N})$ theory. According to Dvali, if there are many species of particles, each mass of particles should be on the order of $1/\sqrt{\mathcal{N}^2}$ [19]; therefore, they are effectively massless.

If $D = 5$, and if the fifth dimension is compactified with size r_c , one can derive the Randall-Sundrum scenario [12]. The basic action is

$$S = S_{gravity} + S_{brane} + S_{brane'}, \quad (4.3)$$

where

$$S_{gravity} = \int dx^4 \int dy \sqrt{-G} \{-\Lambda + 2M_5^3 R\}, \quad (4.4)$$

where G is the determinant of the metric, Λ is the cosmological constant, and M_5 is the Planck mass of 5-dimensions; and

$$S_{brane} = \int dx^4 \{V_{brane} + \mathcal{L}_{brane}\}. \quad (4.5)$$

Then, we naturally obtain

$$M_5^3 \sim g^{-2} \sim \frac{1}{G_5} \sim \mathcal{N}. \quad (4.6)$$

According to Randall and Sundrum [12], one may assume the warped metric ansatz

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \quad (4.7)$$

with the following conditions:

$$V_{brane} = -V_{brane'} = 24M_5^3 k \quad (4.8)$$

and

$$\Lambda = -24M_5^3 k^2. \quad (4.9)$$

If one assumes there is a $r_c \rightarrow \infty$ limit, we still have a finite Planck mass:

$$M_{Pl}^2 = \frac{M_5^3}{k} \left[1 - e^{-2kr_c\pi} \right] \rightarrow \frac{M_5^3}{k}. \quad (4.10)$$

In this setup, the bulk is an anti de Sitter space. However, if one couples the theory with a scalar field with a potential, a de Sitter space can be derived. We follow the results of a thorough paper of Shiromizu, Maeda, and Sasaki [13]. From the 5-dimensional Einstein equation, one can derive its 4-dimensional part. After assuming the metric ansatz

$$ds^2 = d\chi^2 + q_{\mu\nu} dx^\mu dx^\nu \quad (4.11)$$

and the energy momentum tensor

$$T_{\mu\nu} = -\Lambda g_{\mu\nu} + \delta(\chi)(-\lambda q_{\mu\nu} + \tau_{\mu\nu}), \quad (4.12)$$

one can impose Z_2 symmetry along the χ direction. Here, λ and $\tau_{\mu\nu}$ are the vacuum energy and the energy momentum tensor of the brane world.

Then, finally, one can derive the 4-dimensional Einstein equation:

$$^{(4)}G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + 8\pi G_4 \tau_{\mu\nu} + \frac{1}{M_5^6} \pi_{\mu\nu} - E_{\mu\nu}, \quad (4.13)$$

where

$$\Lambda_4 = \frac{1}{2M_5^3} \left(\Lambda + \frac{\lambda^2}{6M_5^3} \right), \quad (4.14)$$

$$G_4 = \frac{\lambda}{48\pi M_5^6}, \quad (4.15)$$

$$\pi_{\mu\nu} = -\frac{1}{4} \tau_{\mu\alpha} \tau_\nu^\alpha + \frac{1}{12} \tau \tau_{\mu\nu} + \frac{1}{8} q_{\mu\nu} \tau_{\alpha\beta} \tau^{\alpha\beta} - \frac{1}{24} q_{\mu\nu} \tau^2, \quad (4.16)$$

and $E_{\mu\nu}$ is the 5-dimensional Weyl tensor. Note that $\pi_{\mu\nu}$ and the longitudinal part of the Weyl tensor is on the order of τ^2 ; if $\tau \sim R \ll 1$, one can neglect them (R is the 4-dimensional curvature). Also, the transverse part of the Weyl tensor is negligible as long as $rk \gg 1$, where $k = \lambda/(6M_5^3)$ for a $\Lambda_4 \simeq 0$ case. If we can apply two conditions, the following can be obtained:

$$^{(4)}G_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + 8\pi G_4 \tau_{\mu\nu}. \quad (4.17)$$

Now, let us check whether the theory allows the conditions of re-scalings or not. If there is a sufficient number of D3-branes, the theory on the brane will have $SU(\mathcal{N})$, and the theory contains massless fields on the order of $N \sim \mathcal{N}^2$. Of course, the semi-classical effects (the Hawking radiation) are dominant on branes; thus, Equation (4.17) naturally induces Equation (3.6) in a semi-classical sense.

If one chooses

$$\lambda \sim \mathcal{N}, \quad (4.18)$$

then k is on the order of 1, and G_4 is on the order of $1/\mathcal{N} \sim g^2$. Now, one can check the consistency of Equation (4.17). If the gravitational constant is re-scaled in the $\Lambda_4 \simeq 0$ limit, the following can be obtained:

$$^{(4)}G_{\mu\nu} = 8\pi \tau_{\mu\nu} + \pi_{\mu\nu} - E_{\mu\nu}. \quad (4.19)$$

However, after the re-scaling, the terms of order τ^2 will be quickly become smaller and smaller on the order of $1/N^2$. Also, since $k \sim 1$, if we choose the characteristic size of the black hole to be sufficiently large, $rk \gg 1$ can be easily obtained. Therefore, our setup is consistent with Equation (4.17).

Therefore, in this brane world setup, we can find a correspondence between the semi-classical equation (3.7) and the brane world setup; if there is a solution from Equation (3.7), there exists a brane world universe with a sufficiently large \mathcal{N} that the same causal structure is justified.

4.2 Brane world: small black holes

Now let us look at small black holes in the brane world; if we choose a small cosmological constant limit

$$|\Lambda| \ll 1, \quad (4.20)$$

then

$$k^2 \lesssim \frac{1}{\mathcal{N}} \quad (4.21)$$

holds. Here, small means that the size of black hole is smaller than the characteristic size ($\sim 1/\sqrt{-\Lambda}$) of the bulk anti de Sitter space.

In small black holes, one cannot assume that the physics is confined in 4-dimensional gravity. Since one can assume that almost all Hawking radiation emits along the brane modes, we can suggest the following Einstein equation

$${}^{(5)}G_{\mu\nu} = 8\pi {}^{(5)}T_{\mu\nu} + 8\pi {}^{(5)}\langle T_{\mu\nu} \rangle, \quad (4.22)$$

where ${}^{(5)}T_{\mu\nu}$ is the energy-momentum tensor of 5-dimensions and ${}^{(5)}\langle T_{\mu\nu} \rangle = \delta(\chi)\langle T_{\mu\nu} \rangle$ is the re-normalized stress tensor for brane modes. If L, M , and T are solutions of the equation, then $L' = L(G_5 h)^{1/3}$, $M' = M(h^{2/3}/G_5^{1/3})$, and $T' = T(G_5 h)^{1/3}$ are solutions of the following equation:

$${}^{(5)}G_{\mu\nu} = 8\pi G_5 {}^{(5)}T_{\mu\nu} + 8\pi G_5 h {}^{(5)}\langle T_{\mu\nu} \rangle, \quad (4.23)$$

since ${}^{(5)}G_{\mu\nu}$ has a dimension L^{-2} , ${}^{(5)}T_{\mu\nu}$ has a dimension ML^{-4} , and ${}^{(5)}\langle T_{\mu\nu} \rangle$ has a dimension L^{-5} . If there are N independent modes for the Hawking radiation, we can choose $h = N\hbar$. Therefore, even if we do not assume 4-dimensions, one can apply the same re-scaling.

As a dimensional analysis, this scheme can be extended for arbitrary dimensions; if D -dimensions are not compactified and the other dimensions are compactified as in the Randall-Sundrum scenario, we can assume the following:

$$M_D^{D-2} \sim g^{-2} \sim \frac{1}{G_D} \sim \mathcal{N}. \quad (4.24)$$

Also, we apply the next condition for small black holes:

$$\Lambda \sim -M_D^{D-2} k^{D-3} \ll 1. \quad (4.25)$$

Then

$$k \lesssim \mathcal{N}^{-1/(D-3)} \quad (4.26)$$

is reasonable. If L, M , and T are solutions of

$${}^{(D)}G_{\mu\nu} = 8\pi {}^{(D)}T_{\mu\nu} + 8\pi {}^{(D)}\langle T_{\mu\nu} \rangle, \quad (4.27)$$

then

$$\begin{aligned} L' &= L(G_D h)^{\frac{1}{D-2}}, \\ M' &= M \left(\frac{h^{\frac{D-3}{D-2}}}{G_D^{\frac{1}{D-2}}} \right), \\ T' &= T(G_D h)^{\frac{1}{D-2}}, \end{aligned} \tag{4.28}$$

are solutions of a proper N limit, if one chooses $h = N\hbar$.

Therefore, all benefits outlined in section 3.3 can be applied to small black holes of the brane world in the large \mathcal{N} limit.

5. Applications

5.1 Schwarzschild black holes

In general, $(4+n)$ -dimensional Schwarzschild black holes are described by the following metric:

$$ds^2 = - \left(1 - \frac{\mu}{r^2} r^{1-n} \right) dt^2 + \frac{1}{\left(1 - \frac{\mu}{r^2} r^{1-n} \right)} dr^2 + r^2 d\Omega^2, \tag{5.1}$$

where μ is a parameter that is related to mass and Ω is a solid angle of a $(n+2)$ -sphere.

The horizon is $r_0 = \mu^{1/(n+1)}$, and it is not difficult to confirm that the Hawking temperature is on the order of $T \sim 1/r_0$. Therefore, if one considers the lifetime of a black hole for a $(4+n)$ -dimensional black hole, one can easily calculate that

$$\frac{dM}{dt} \sim \frac{d\mu}{dt} \sim A_{(4+n)} T^{(4+n)} \sim r_0^{2+n} \frac{1}{r_0^{4+n}} \sim \frac{1}{r_0^2}, \tag{5.2}$$

and one obtains a lifetime $\tau \sim \mu^{\frac{n+3}{n+1}} \sim r_0^{n+3}$ [14]. For a small black hole of $(4+n)$ -dimensions in the brane world, one needs to calculate the brane modes $A_4 T^4$ for the mass loss, but the conclusion will be the same.

For a 4-dimensional case, by the re-scaling, we will re-scale all length, mass, and time parameters by \sqrt{N} . Now, the lifetime is re-scaled by

$$\tau \sim M^3 \rightarrow \frac{(\sqrt{N}M)^3}{N} \sim \sqrt{N}M^3 \sim \sqrt{N}\tau. \tag{5.3}$$

This conclusion is the correct interpretation, since τ has a time dimension. Here, we have to divide the lifetime by N , since there are (effectively) N -independent fields to use the Hawking radiation.

For a 5-dimensional case, the gravitational constant G_5 has a dimension of $L^4 M^{-1} T^{-2}$ and all length and time parameters will be re-scaled by $N^{1/3}$; mass parameters will be re-scaled by $N^{2/3}$ (as given by Equation (4.28)). The horizon is $r_0 \sim \sqrt{\mu}$. Therefore,

$$\tau \sim r_0^4 \rightarrow \frac{(N^{1/3}r_0)^4}{N} \sim N^{1/3}r_0^4 \sim N^{1/3}\tau. \tag{5.4}$$

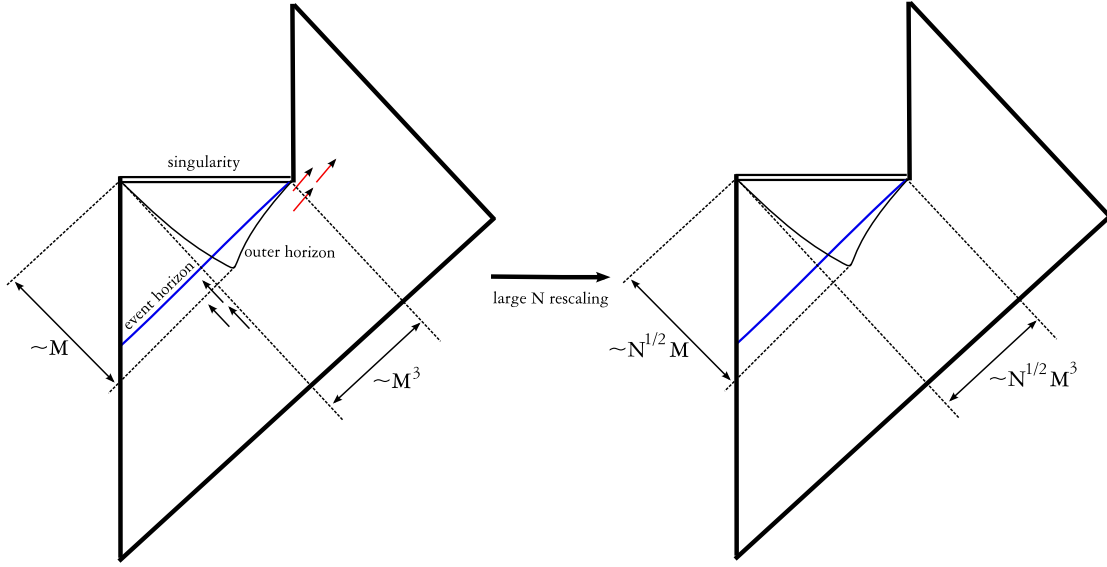


Figure 2: The causal structure of a Schwarzschild black hole.

Again, we have a correct dimensional analysis.

These are consistent results. Since our re-scalings conserve relative distances between points, we will have identical causal structures. And the relative location of the information retention time will be invariant on the Penrose diagram (Figure 2).

For the next calculation, we will comment on a simple extension to Kruskal-Szekeres coordinates [6][20]. Let us assume to neglect the angular part, and we assume the form

$$ds^2 = F(R) (-R^2 d\omega^2 + dR^2). \quad (5.5)$$

To compare the original metric, the following assumptions are reasonable:

$$\begin{aligned} d\omega^2 &= \frac{dt^2}{r_0^2}, \\ R^2 F(R) &= r_0^2 \left(1 - \frac{\mu}{r^2} r^{1-n} \right), \\ F(R) dR^2 &= \frac{1}{\left(1 - \frac{\mu}{r^2} r^{1-n} \right)} dr^2. \end{aligned} \quad (5.6)$$

In terms of the coordinate R , the singularity occurs at $R^2 = -r_0^2$; and the horizon occurs at $R = 0$.

Now, we can choose another metric and coordinates (U, V) by

$$\begin{aligned} V &= R e^\omega, \\ U &= -R e^{-\omega}, \\ ds^2 &= -F(R) dU dV. \end{aligned} \quad (5.7)$$

Here, the singularity is $UV = r_0^2$.

Now, we can state the condition of a duplication experiment of black hole complementarity in a Schwarzschild black hole [7]. The first observer falls into a black hole, and sends a signal to the out-going direction around time Δt . Now assume that a second observer hovers above the horizon at a distance of the order of the Planck length, and jumps into the black hole at the information retention time $\sim \tau$. Then, the initial location of the second observer is $V = Re^\omega$, where $R \sim l_{Pl}$ and $\omega \sim \tau/r_0$. Before touching the singularity, the second observer will spend time (in terms of U) around $\sim r_0^2/V$, since the singularity is $UV = r_0^2$. Therefore, the first observer should send a signal around time $\Delta t \sim e^{-\tau/r_0}$.

Therefore, in general, in the $N = 1$ limit, the duplication may be observed if one can send a signal between the time

$$\Delta t \sim \exp -\frac{\tau}{r_0}, \quad (5.8)$$

where τ is a meaningful time scale, e.g., (in 4-dimensions) the information retention time ($\sim M^3$) [5] or the scrambling time ($\sim M \log M$) [21]. Then, in a large N universe, the time scale becomes

$$\Delta t \sim \sqrt{N} \exp -\frac{\tau}{r_0}. \quad (5.9)$$

From the uncertainty relation, the required energy becomes

$$\Delta E \sim \frac{1}{\sqrt{N}} \exp \frac{\tau}{r_0}, \quad (5.10)$$

and since the consistency of complementarity requires $\Delta E > \sqrt{N}M$, the consistency condition becomes

$$\exp \frac{\tau}{r_0} > NM. \quad (5.11)$$

Of course, this condition can be violated by assuming a sufficiently large N [10]. Note that, we can apply this scheme to not only large black holes, but also to small black holes in the brane world.

In this sense, the black hole complementarity principle can be violated even if we consider a Schwarzschild black hole.

5.2 Charged black holes

In some previous papers, semi-classical Einstein equations have been solved via a numerical approach. One of the important motivations was a charged black hole [22][23][24][16]. The classical solution (Reissner-Nordstrom solution) of a charged black hole was well known in general relativity. This solution contains a Cauchy horizon, and its instability was predicted by many authors: so-called mass inflation [25]. However, if there is indeed mass inflation, then its back reaction on the metric cannot be calculated from the Reissner-Nordstrom solution; thus some numerical calculations are inevitable [22][23]. There have been some pioneering works [22][23][24], including by the present authors and colleagues [16] in this area; the authors and colleagues included the re-normalized stress tensor, and solved equations in a consistent manner.

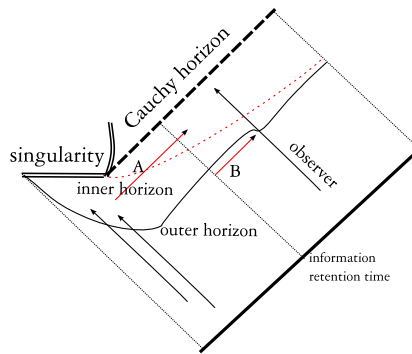


Figure 3: The causal structure of a charged black hole.

After calculations, we observed some novel features. If there is no Hawking radiation, there is a null Cauchy horizon when the advanced time v becomes ∞ [26][22]. However, if there is Hawking radiation, the inner (apparent) horizon bends space-like direction, and approaches the outer (apparent) horizon [24] (Figure 3). Therefore, there will be a space between the Cauchy horizon and the inner horizon [16].

However, a difficult problem arises in interpreting our results. If there is no Hawking radiation, we can assume that $\hbar = 0$, and there are no Planck units and a curvature singularity happens at $R \rightarrow \infty$. However, if there is Hawking radiation, for $\hbar = 1$, there are the Planck units, and a curvature singularity occurs at $R \sim 1$. And we observed that the space between the inner horizon and the Cauchy horizon has large curvatures in general; therefore, we cannot justify the causal structure of there in the semi-classical analysis.

However, in the large N limit, we can apply re-scalings. Therefore, one can conclude that, in a possible universe that can be allowed from string theory, one may trust a region between the inner horizon and the Cauchy horizon. Then the causal structure will violate strong cosmic censorship and black hole complementarity [16].

5.3 How large N is required?

Note that, in a Schwarzschild black hole of 4-dimensions, we can suggest two meaningful time scales: the information retention time $\tau \sim M^3$ [5] and the scrambling time $\tau \sim M \log M$ [21]. Here, the required N is on the order of $\exp \tau/M$. Therefore, in terms of the information retention time, $\exp M^2$ fields are required; whereas in terms of the scrambling time, ~ 1 fields are required. Of course, the latter is overestimated, and it is reasonable to assume that it requires $\sim M^\alpha$, i.e., the polynomial order of M .

In a charged black hole case, the required N is on the order of the calculated curvature R beyond the inner horizon, since the physical curvature R/N should be less than 1. The characteristic scale of R is on the order of the mass function m , which in turn is on the order of the mass inflation factor $\exp \kappa_i(u+v)$ [25], where κ_i is the surface gravity of the inner horizon and u and v are retarded and advanced time parameters. Then, κ_i is on the order of $1/M$ and u or v are on the order of the lifetime M^3 . Therefore, again, the required N is on the order of $\exp M^2$.

One question is whether our real universe violates black hole complementarity or not. It is certain that we need an exotic setup of large N fields to break black hole complementarity which does not correspond with our observable universe. Hence, black hole complementarity is working practically and effectively in the usual setup. However, it could not hold in our semi-classical setup and provide a fundamental resolution of the information loss problem.

5.4 Locality bound in large N limit

Traditionally, there are mainly three options with respect to the information loss problem [27]: (1) information is attached by Hawking radiation, (2) there remains a remnant of very long lifetime, and (3) one cannot regain information from a black hole. If one does not choose (3), then the remaining possibilities are (1) and (2). In terms of semi-classical gravity, however, (1) cannot be obtained [27]. Therefore, one needs to violate some assumptions of semi-classical gravity; the easiest way is to violate locality [28]. However, if one considers the violation of locality only for the area near a singularity, it cannot be helpful to obtain (1). Therefore, if one wants to obtain (1), we expect that there must be an effect of violation of locality for large black holes, which is apparently semi-classical. Note that, (1) or violation of locality naturally implies black hole complementarity [28]; also, black hole complementarity implies a violation of locality [4].

The violation of locality should be related to a strong gravitational effect. Giddings tried to quantify this strong gravitational effect [28]. Let us assume that, for example, two particles are generated from a gravitational background. Each particle has approximately position (x, y) and momentum (p, q) in the center of mass frame. The suggested locality bound is then

$$|x - y| \gtrsim |p + q|, \quad (5.12)$$

for 4-dimensions. If this does not hold, then one may interpret that the gravitational effect is sufficiently strong and violates locality.

Let us assume the creation of two particles with positions (x, y) and momentum (p, q) where the locality bound does not hold. However, the positions and momentum should be solutions within the background metric, and they could be re-scaled by a large N . Since the locality bound relation is scaled by \sqrt{N} for both the left and right hand sides, the direction of the inequality is not changed by the re-scaling. However, we know that the gravitational effect becomes smaller and smaller in the large N limit. Therefore, a reasonable interpretation is that a certain event that violates locality bound does not occur in the large N limit of a large black hole background.

This conclusion is consistent with previous subsections. Giddings suggested that the effect of violation of locality will be dominant by the time of the order of the information retention time M^3 [29] or the scrambling time $M \log M$ [28]. These times are meaningful for an asymptotic observer, but not meaningful for a free-falling observer. For example, if two observers can communicate with each other, then the scrambling near a horizon is meaningless, since the free-falling observer does not be scrambled near the horizon. Of

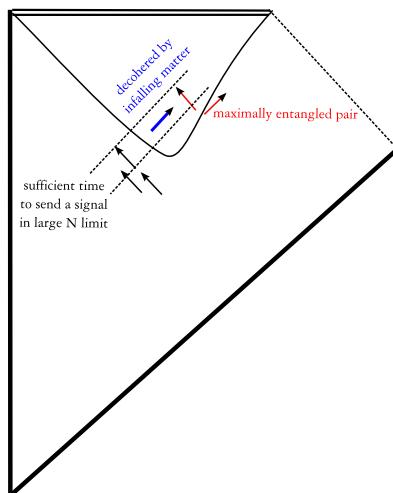


Figure 4: Arbitrary decoherence can be implemented if there are sufficient N .

course, in a small N universe, according to arguments of black hole complementarity, two observers could not communicate. However, in a large N limit, we know that they can communicate freely. Therefore, it is a consistent interpretation that, in the large N limit, these time scales are meaningless and the locality bound must not be violated with a large black hole background.

The violation of locality for semi-classical black holes then cannot be a fundamental resolution of the information loss problem. Of course, it may be helpful in understanding certain small N black holes with certain causal structures; but it cannot be applied to explain unitarity of large N black holes with the same causal structures. For a given causal structure, and for all possible N , if the violation of locality is essential to explain unitarity, then the violation should be near the singularity.

5.5 Is an entanglement helpful to the information loss problem?

As another approach, we can turn to ideas of quantum information theory. This may be helpful to address the information loss problem as well as black hole complementarity [30][31].

For example, a proposal of Horowitz and Maldacena [32] assumes a final state of the singularity. They assume that the Hawking radiation is maximally entangled between the in-going and the out-going part. One potential concern here is that the in-falling matter may destroy the entanglement between the inside and the outside [33]. If this is true, then the proposal will not hold [34]. However, it remains unclear whether interactions between the in-falling matter and the in-going Hawking radiation can be implemented to violate the proposal. If there is a kind of limitation to use the interactions, then one may state that even if there is a potential problem regarding maximal entanglement, it will work in real situations.

In our re-scaling setup, we use the same Penrose diagram as the number of massless

fields grows. Here, since a length of any two points on the Penrose diagram becomes longer and longer by a factor \sqrt{N} , one can send a signal between arbitrary two points on the Penrose diagram with reasonably small energy. This implies that we can destroy the entanglement of the proposal as desired (Figure 4). Of course, more concrete discussion about this issue is necessary; nevertheless, a naive expectation is that, if one assumes a large N , one can destroy the entanglement of Hawking radiation, and this will break the core assumption of the proposal. Hence, it is unclear whether a quantum information theoretical resolution of the information loss problem is a fundamental resolution.

5.6 Singularity and remnant picture

What will happen if black hole complementarity is not true?

Let us assume that an ideal observer who can control all outcomes of a black hole can reconstruct the original information in principle. Then, if the area is proportional to the physical entropy of a black hole, black hole complementarity is inevitable from Page's argument [5]; information should escape around the information retention time, and even in this time, a black hole can be sufficiently large. Therefore, information should be attached by Hawking radiation, and there can be two ideal observers, where one is an in-falling observer and the other is an out-going observer. If the no cloning theorem is correct, then they should not compare observations. However, as we discussed, in the large N universe, black hole complementarity cannot be true, and the violation of locality for large black holes cannot be helpful for information conservation.

Then which assumption is invalid in the previous picture? If unitarity is correct, there remains two possibilities.

1. There is no such ideal observer who can control all outcomes of a black hole [35].
2. The area is not the physical entropy but just an apparent entropy; therefore, the real entropy should be calculated by the inside degrees of freedom, or calculation of real physical entropy is meaningless [36][10].

Note that, since the first possibility implies that the outside observer cannot reconstruct the information from a black hole, if it is correct, then the assumption that the physical entropy is proportional to its area becomes meaningless. Thus, if the first possibility is correct, then the second possibility should be correct, too. Now, it is inevitable to accept the second possibility.

Therefore, first, let us assume the negation of the first possibility and assume the second possibility; that is, the real information retention time is not the moment when the initial area of a black hole decreases to its half value. Hence, information will not be contained by the Hawking radiation, and all information should be contained by the final remnant. Then the final remnant, which has in general small area, should contain very large entropy; then, in general, the outcome of a small remnant should have very large entropy, and the wavelength of the outcome of the final remnant then should be very long if the outcomes contain all initial information [27][37]. Therefore, it is equivalent to the remnant picture, where the remnant has a very long lifetime.

We will not consider details of this idea; however, one important comment is that, to study this possibility, it is necessary to solve the problem of singularity. In black hole complementarity, it is not necessary to consider the trouble of singularity; however, in the remnant picture, one needs to calculate the entropy of the final remnant, and full calculations around the singularity are required. If full calculation around the singularity is possible, then one can extend the causal structure beyond the singularity. Some authors have contended that the information loss problem can be resolved if the causal structure beyond a singularity is solved [38] (we call this the causal structure picture). This assertion is partly correct, but not entirely; they did not consider a problem of entropy.

In conclusion, we can suggest a very cautious but probable comment on the information loss problem. One possibility is that there is no ideal observer who can reconstruct the original information; the other possibility is that information is retained by a long lifetime remnant. For the latter idea, one needs to study the entropy near a singularity as well as the causal structure beyond the singularity. As the authors understand, the regular black hole picture ([38] and references therein) or the causal structure picture [39] are equivalent with the remnant picture; the former models will inherit the same problems, if the remnant picture has problems of entropy.

6. Discussion: Toward singularity

We claim that large N semi-classical gravity is a useful tool to examine the information loss problem. We can define re-scaling between a $N = 1$ universe and a large N universe. Also, we can find a realization of large N setup from the brane world scenario of string theory. Here, re-scaling by large N fields can preserve the causal structure of any semi-classical black hole solution in the $N = 1$ universe.

If a suggested resolution of the information loss problem must be for the final stage of the black hole, the present paper is not relevant. However, if the resolution could be applied to even a semi-classical black hole, then our discussion can be meaningful. Any idea resolving the information loss problem should be valid in a large N setup. On these grounds, it is possible to test the consistency of black hole complementarity and violation of locality for semi-classical black holes; black hole complementarity and violation of locality do not hold in the large N limit.

Thus, we suspect that the essence of the information loss problem may be located around the singularity at the final stage of the black hole evolution, where a new perspective on the causality is required and hence the semi-classical argument is no longer valid.

The conclusion is very simple. If one assumes a large N limit, semi-classical gravity should be valid. Then it is clear that the semi-classical theory contains the information loss problem. Of course, a traditional idea is that, if we know the quantum gravity, the paradox will be resolved. However, what is the role of the quantum gravity? If a certain resolution argues that the information loss problem can be resolved by arguments on large semi-classical black holes alone, we can say that the resolution cannot be correct in the large N limit, where the semi-classical calculations become correct. Therefore, one sound conclusion is that *the information loss problem will be resolved by using an idea that resolves*

the problem of the singularity; in other words, if it does not resolve the singularity, it cannot be a fundamental resolution to the information loss problem.

Acknowledgments

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